(c) $h'(x) = f(x)$

$h'(x) = f(x) = 0$, when $x = 0$ and $x = 2$.

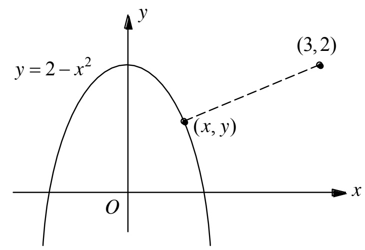
h has a relative maximum at $x = 0$, because h' changes from positive to negative at $x = 0$.

h has a relative minimum at $x = 2$, because h' changes from negative to positive at $x = 2$.

(d) The graph of h has a point of inflection at $x = 1$ because $h'' = f'$ changes sign at $x = 1$.

3.7 Optimization Problems

1. D Use a graphing calculator to sketch the graph of $y = 2 - x^2$.



The distance between the point $(3,2)$ and a point (x, y) on the graph $y = 2 - x^2$ is

$d = \sqrt{(x-3)^2 + (y-2)^2}$

$= \sqrt{(x-3)^2 + ((2-x^2)-2)^2}$ Make substitution, $y = 2 - x^2$.

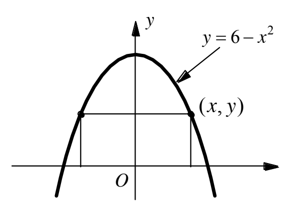
$= \sqrt{(x-3)^2 + (x^2)^2} = \sqrt{x^4 + x^2 - 6x + 9}$

We need only find the critical numbers of $f(x) = x^4 + x^2 - 6x + 9$, because d is smallest when the expression inside the radical is smallest.

$f'(x) = 4x^3 + 2x - 6$. $f'(x) = 0 \Rightarrow x = 1 \Rightarrow y = 1$

The point on the curve nearest to $(3,2)$ is $(1,1)$.

2. E $A = 2xy = 2x(6 - x^2) = 12x - 2x^3$



$\dfrac{dA}{dx} = 12 - 6x^2 = 6(2 - x^2)$. $\dfrac{dA}{dx} = 0 \Rightarrow x = \pm\sqrt{2}$.

$\dfrac{dA}{dx}$ changes from positive to negative at $x = \sqrt{2}$, so the area is maximized when $x = \sqrt{2}$.

$A = 12x - 2x^3 = 12(\sqrt{2}) - 2(\sqrt{2})^3 = 8\sqrt{2}$

3. C $xy = x(\dfrac{1}{\sqrt{x}} - \sqrt{x}) = \sqrt{x} - x\sqrt{x} = x^{1/2} - x^{3/2}$

$\dfrac{d}{dx}(xy) = \dfrac{1}{2}x^{-1/2} - \dfrac{3}{2}x^{1/2} = \dfrac{1}{2\sqrt{x}}(1-3x)$. $\dfrac{d}{dx}(xy) = 0 \Rightarrow x = \dfrac{1}{3}$

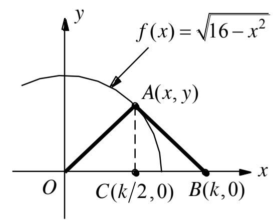
$xy = \sqrt{x} - x\sqrt{x} = \dfrac{1}{\sqrt{3}} - \dfrac{1}{3}\cdot\dfrac{1}{\sqrt{3}} = \dfrac{2}{3\sqrt{3}} = \dfrac{2\sqrt{3}}{9}$

4. E $y' = \dfrac{\sin x(-\sin x) - (\cos x - m)\cos x}{\sin^2 x} = \dfrac{-(\sin^2 x + \cos^2 x) + m\cos x}{\sin^2 x} = \dfrac{-1 + m\cos x}{\sin^2 x}$

$$y'\big|_{x=\frac{\pi}{4}} = \frac{-1+m\cos(\pi/4)}{\sin^2(\pi/4)} = 0 \ \Rightarrow\ -1+m\cos(\frac{\pi}{4}) = 0 \ \Rightarrow\ -1+m\cdot\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow\ m = \sqrt{2}$$

5. (a) Draw $\overline{AC}$, which is perpendicular to the x-axis. $OC = BC$ since ΔOAB is an isosceles triangle.



$$A = \frac{1}{2}k\cdot y = \frac{1}{2}k\cdot\sqrt{16-x^2} \qquad y=\sqrt{16-x^2}$$

$$= \frac{1}{2}k\cdot\sqrt{16-(\frac{k}{2})^2} \qquad x = \frac{k}{2}$$

$$= \frac{1}{4}k\sqrt{64-k^2} \qquad \text{Simplified.}$$

(b) $$\frac{dA}{dk} = \frac{1}{4}\left[k\cdot\frac{1}{2}(64-k^2)^{-1/2}(-2k)+\sqrt{64-k^2}\right] = \frac{1}{4}\left[\frac{-k^2}{\sqrt{64-k^2}}+\sqrt{64-k^2}\right]$$

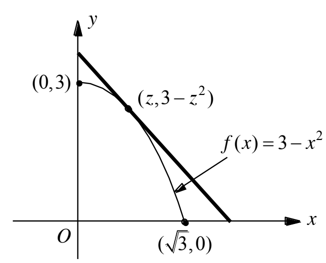
$$= \frac{1}{4}\left[\frac{-k^2+64-k^2}{\sqrt{64-k^2}}\right] = \frac{1}{2}\left[\frac{-k^2+32}{\sqrt{64-k^2}}\right]$$

$$\frac{dA}{dk} = 0 \ \Rightarrow\ k^2 = 32 \ \Rightarrow\ k = 4\sqrt{2}$$

Signs of $\frac{dA}{dk}$: $+$ (increasing) | $-$ (decreasing), at $4\sqrt{2}$

The triangle has a maximum area when $k = 4\sqrt{2}$.

6. (a) $f(x) = 3-x^2$, $f'(x) = -2x$, $f'(z) = -2z$



$$y-(3-z^2) = -2z(x-z) \qquad \text{Point slope form of the line}$$

$$y = -2zx+z^2+3 \qquad \text{Equation of the tangent line}$$

(b) x-intercept of the tangent line is $\dfrac{z^2+3}{2z}$.

y-intercept of the tangent line is z^2+3.

$$A(z) = \frac{1}{2}(\frac{z^2+3}{2z})(z^2+3) = \frac{1}{4}\frac{(z^2+3)^2}{z}$$

$$A'(z) = \frac{1}{4}\frac{z\cdot 2(z^2+3)\cdot 2z-(z^2+3)^2\cdot 1}{z^2} = \frac{3(z^2+3)(z^2-1)}{4z^2}$$

$$A'(z) = 0 \ \Rightarrow\ z^2-1 = 0 \ \Rightarrow\ z = 1$$

Signs of $A'(z)$: $-$ (decreasing) | $+$ (increasing), at 1

The triangle has a minimum area when $z = 1$.